Chapter 20

ALTERNATING CURRENT

A.) The Production of AC:

1.) When a coil is placed in a magnetic field and is rotated, a changing flux $\Delta\Phi_m$ through the coil will produce an induced EMF across the coil's leads. A surprising result is found when Lenz's Law is used to determine the direction of the induced current and, hence, the high and low voltage side of the coil as the coil rotates. Follow along:

a.) Assume the external magnetic field is directed into the page and the coil is initially facing the B-field (see Figure 20.1a--note that the right side of the coil--the part that will initially rotate into the page--is darker; this has been done to make it easier to follow the coil through the complete 360^o rotation). As the coil begins to rotate, the external flux through the coil's face diminishes (Figure 20.1b). The coil generates an induced current which opposes the decrease of flux, which means an induced B-field is created into the page. The direction of current flow required to do this is clockwise, which means lead A must be the positive side of the coil.

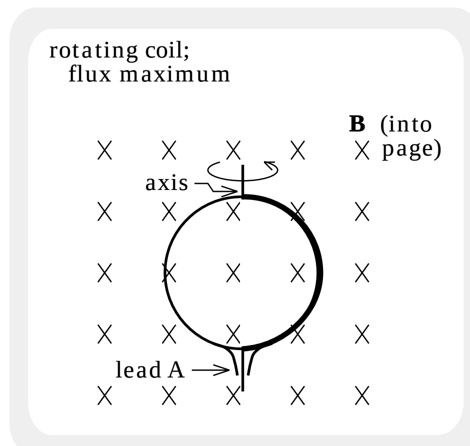

FIGURE 20.1a

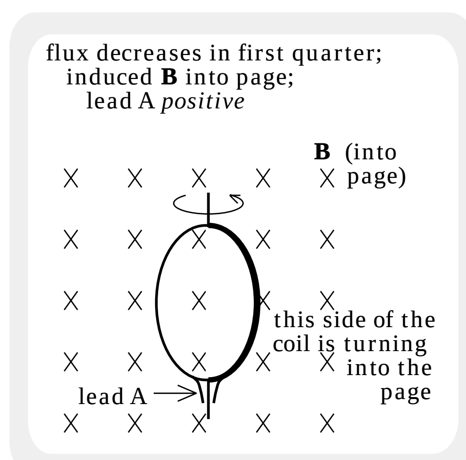

FIGURE 20.1b

b.) The coil rotates through the no-external-flux position and into the second quarter of its cycle (Figure 20.1c on the next page). In this part of its motion, the external flux is increasing. The induced B-field required to oppose this increase must be out of the page, which requires an induced current that is counterclockwise relative to ourselves. As such, lead A must again be the positive side of the coil.

c.) The coil rotates through the maximum-flux position and into the third quarter of its cycle (Figure 20.1d). In this part of its motion, the external flux is decreasing. The induced B-field required to oppose this decrease must be into the page, which requires a current that is clockwise relative to ourselves. As such, lead A must be negative.

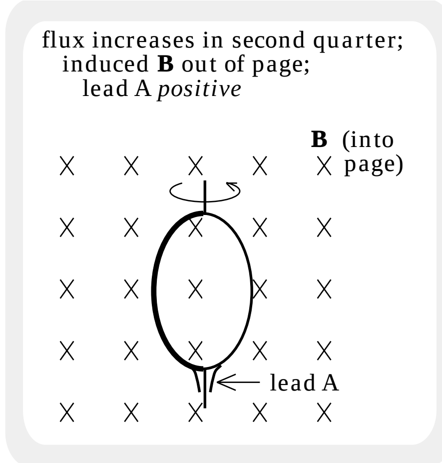

FIGURE 20.1c

d.) Lead A will be negative through the last quarter of the rotation (just as in Part b above, but with the lead positions switched).

e.) Bottom line: The high-voltage side of a rotating coil alternates from one side to the other as the coil turns. This is how alternate current (AC) is generated.

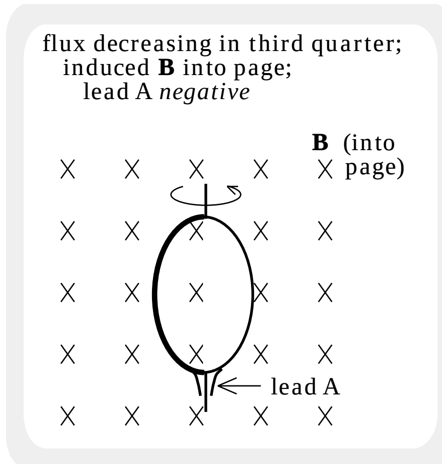

FIGURE 20.1d

2.) AC Power--From the Power Plant to You:

a.) Power production: Consider a hydroelectric plant. A waterfall turns a turbine, the shaft of which is attached to a giant coil. The coil is suspended in a fixed magnetic field. As the coil rotates, AC is produced across its leads. Sliding contacts tap the alternating voltage.

b.) Transport: Sending the electrical power to the city requires the use of wires. The single biggest source of energy loss during this transfer is due to the heating-up of those wires. As high currents generate lots of heat, the trick is to keep the current as low as possible.

Using a step-up transformer, the voltage is stepped up to, say, 50,000 volts. This drops the current down quite low for the span between the power generator and the city. As there are few toasters that can handle 50,000 volts, a step-down transformer steps the voltage at the city down either to 110 or 220 volts AC allowing current-availability to go upward in the process. In that way, power companies can accommodate hundreds of thousands of homes at once.

The entire process is displayed in Figure 20.2 on the next page.

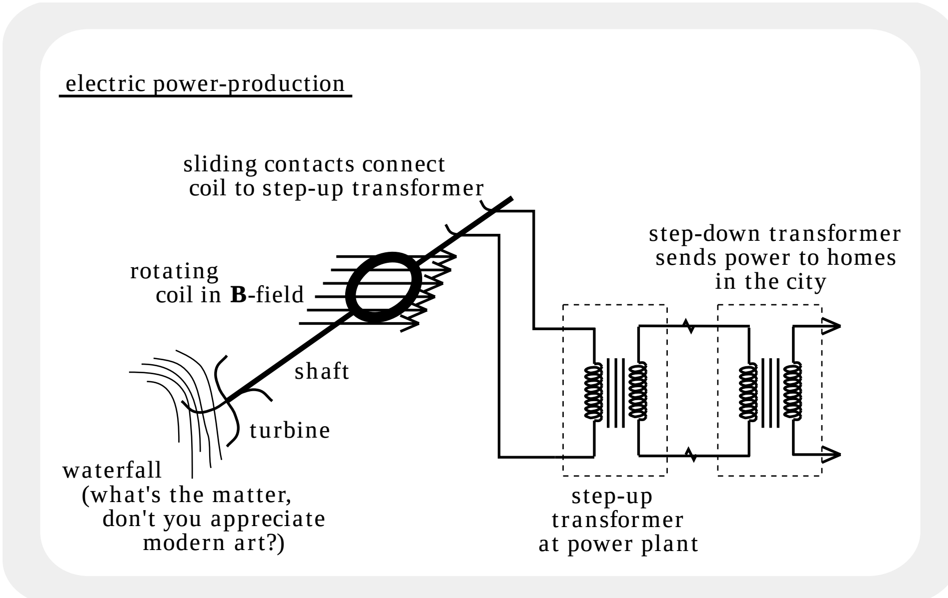

FIGURE 20.2

B.) The Fundamentals of Alternating Currents:

1.) The circuit symbol for an AC power supply is shown in Figure 20.3.

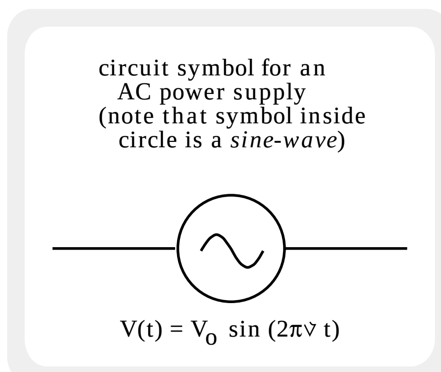

FIGURE 20.3

2.) Figure 20.4 graphs the voltage difference between the two terminals as a function of the time-related variable ωt. Assuming the voltage difference across the terminals is positive when the left terminal is the higher voltage, we find:

a.) At t = 0 there is no voltage difference between the terminals.

b.) As time progresses the voltage difference between the two terminals gets larger until it hits some maximum (see Figure 20.5a on the next page). The maximum is defined as the amplitude of the voltage function and is sometimes called the

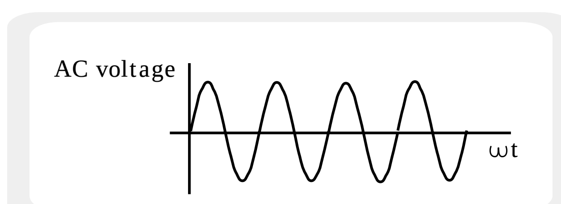

FIGURE 20.4

peak voltage. Under most circumstances this peak voltage is symbolized as V_o.

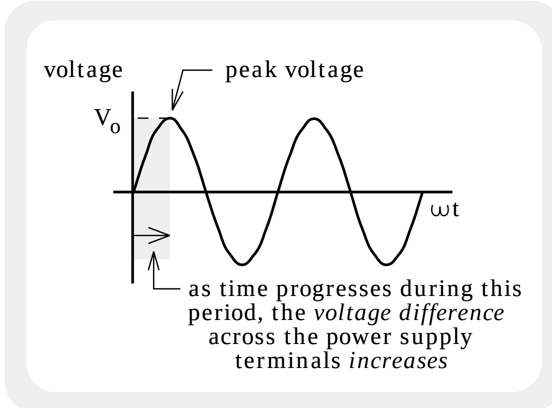

FIGURE 20.5a

c.) After reaching its peak, the voltage difference between the terminals decreases back down to zero (see Figure 20.5b). At this point, the high and low voltage terminals reverse. To denote this change, reverse-polarity voltages are defined as negative voltages.

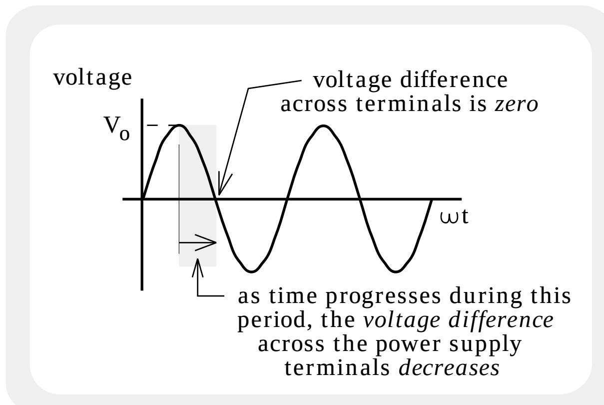

FIGURE 20.5b

Note: The implication is that there is no real difference between a positive or negative voltage when dealing with AC. In one case the high voltage terminal is, maybe, on the left while in the other case the high voltage terminal is on the right. The words "positive" and "negative" are used simply to distinguish between the two situations.

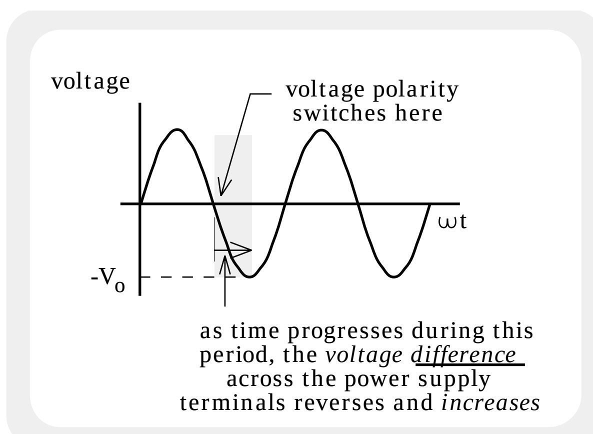

FIGURE 20.5c

d.) After polarity reversal, the voltage difference between the terminals gets larger with time (Figure 20.5c) until it again reaches its maximum negative peak V_o ($-V_o$ on the graph--see Figure 20.5c). It then proceeds back towards zero. Once at zero, the polarity changes and the process starts all over again.

e.) The function that defines this behavior is a sine wave. That is, time-dependent voltage sources can be mathematically characterized as:

$$V(t) = V_o \sin(\omega t)$$
$$= V_o \sin(2\pi\nu t),$$

where ω is the angular frequency in radians per second of the alternating voltage and ν is the more common frequency variable in cycles per second or hertz.

Note: The AC in your home has a frequency of 60 hertz. Multiplying by 2π yields a voltage function for home wall-sockets of: $V(t) = V_o \sin(377\ t)$.

3.) The current through a resistor hooked to an AC power supply produces a graph similar to that shown in Figure 20.4. That is, the time-dependent current function looks just like the time-dependent voltage function, or:

$$i(t) = i_o \sin(\omega t)$$
$$= i_o \sin(2\pi\nu t).$$

Note: For resistors, the frequency of the AC voltage source and the frequency of the current functions will always be the same.

C.) RMS Voltages and Currents:

1.) AC voltages and the currents they produce are constantly varying not only in magnitude but also in direction. It would be nice to quantify such AC variables with some kind of time-INDEPENDENT, pseudo-average value ("pseudo" in the sense that a normal average will not work--the time-average of a sine wave is zero). As such, a bit of a different view has been taken. Consider the following:

a.) Hook a DC power source to a resistor (Figure 20.6a). The amount of power dissipated by the resistor will be i^2R, where i is the constant DC current in the circuit.

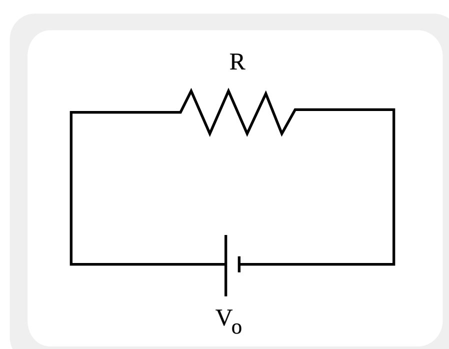

FIGURE 20.6a

b.) Hook an AC power source to the same resistor (Figure 20.6b). There will be power dissipated by the resistor, but how much? To determine this:

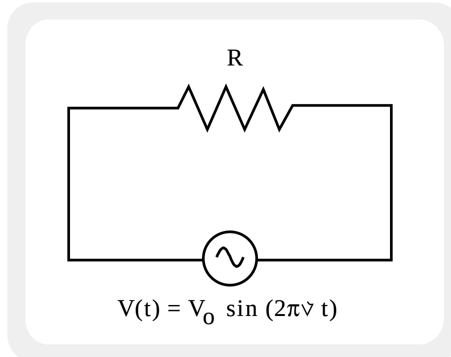

FIGURE 20.6b

i.) At any instant t, the amount of power being dissipated by a resistor will be i^2R, where i is the circuit's current at time t. Over an extended period of time, the power dissipated will be $(i^2)_{avg}R$. In other words, the average of the current squared is the important quantity here.

ii.) AC current varies as shown in Figure 20.7. The AC current squared is also shown in that figure.

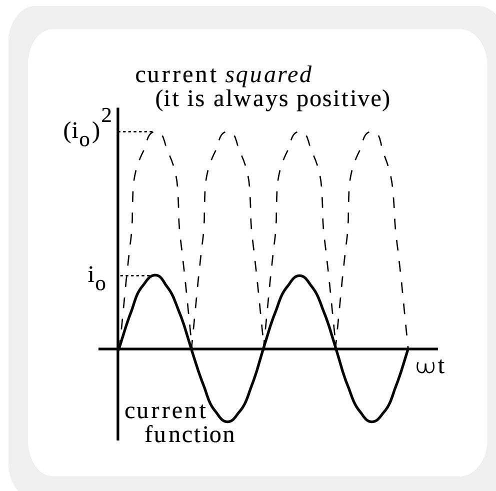

FIGURE 20.7

iii.) To a good approximation, the average of the AC current squared is $i_o^2/2$. The power being dissipated by the resistor will, therefore, be:

$$P = (i^2)_{avg}R$$
$$= (i_o^2/2)\,R.$$

iv.) If we want to define a constant, time-independent current value--an effective current i_{eff}--that when squared and multiplied by R will equal the amount of power dissipated by the resistor when in the AC circuit, we can write:

$$(i_{eff})^2\,R = (i^2)_{avg}R$$
$$= (i_o^2/2)\,R.$$

This implies that:

$$i_{eff} = i_o / (2)^{1/2}$$
$$= .707\, i_o.$$

v.) Bottom line: The power-producing current provided to a circuit by an AC source has a DC equivalent. That DC equivalent (it was called the effective current above) is equal to $.707i_o$, where i_o is the amplitude of the AC current function.

Because this value was determined by taking the square root of the mean value of the square of the AC current, the value is called the "root mean square" value. In almost all physics textbooks, this is shortened to RMS. In other words, to be conventional, the effective current should be termed i_{RMS}.

vi.) Summary: The effective current in an AC circuit is called the RMS current and is equal to:

$$i_{rms} = .707i_o.$$

Similarly, the effective voltage in an AC circuit is called the RMS voltage and is equal to:

$$V_{rms} = .707V_o.$$

c.) All AC meters read RMS values (both ammeters and voltmeters).

i.) Example: You are probably aware that most of the wall sockets in your home provide 110 volts AC at a frequency of 60 hertz (the few that do not are rated at 220 volts AC and are hooked to big appliances like washers and dryers). This means that if we plug the leads of an AC voltmeter into a wall-socket, the meter will read a constant RMS voltage of 110 volts.

With that in mind, write the function that characterizes a wall-socket's voltage as a function of time.

Solution: As stated, you know that the RMS voltage reading is 110 volts. The relationship between voltage amplitude V_o and RMS voltage V_{RMS} is:

$$\begin{aligned} V_{rms} &= .707\ V_o \\ \Rightarrow \quad V_o &= V_{rms}/.707 \\ &= (110\text{ volts})/(.707) \\ &= 155.6\text{ volts}. \end{aligned}$$

With $\nu = 60$ hz, the wall-socket voltage function is:

$$V(t) = V_o \sin(2\pi\nu t)$$
$$= 155.6 \sin(377 t).$$

Note: Another voltage value sometimes cited is the peak-to-peak (more accurately, the peak-to-trough) voltage. By definition, $V_{pp} = 2V_o$.

D.) A Lone Resistor in an AC Circuit:

1.) We have already established that in DC circuits, the voltage across a resistor is directly proportional to the current through the resistor. Mathematically, this relationship is summed up in Ohm's Law which states that $V_R = iR$. The proportionality constant R is a relative measure of the resistive nature of the circuit. That is, a large R implies a lot of resistance to charge flow whereas a small R implies very little current resistance.

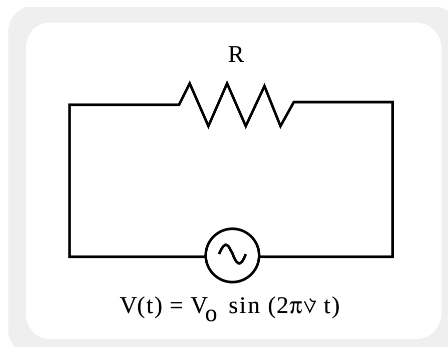

FIGURE 20.8

2.) A resistor in an AC circuit (see Figure 20.8) behaves just as a resistor does in a DC circuit. At any instant, the voltage and current are related by Ohm's Law ($V_{rms} = i_{rms} R$) and the power is related as $P = (i_{rms})^2 R$.

3.) A power supply that can generate a variable frequency voltage (i.e., voltages at frequencies other than 60 hertz) is called a sine wave generator or a function generator.

4.) No matter what frequency a sine wave generator outputs into a resistor circuit, the resistive nature of the resistor will always be the same. Put another way, the resistive nature of a resistor is not frequency-dependent.

5.) The current through a resistor and the voltage across a resistor will always be in phase. That is, as the voltage increases, so will the current. As the voltage decreases, so will the current. When the voltage is zero, so is the current, etc.

E.) An Inductor and Resistor in an AC Circuit:

1.) Consider the RL circuit shown in Figure 20.9.

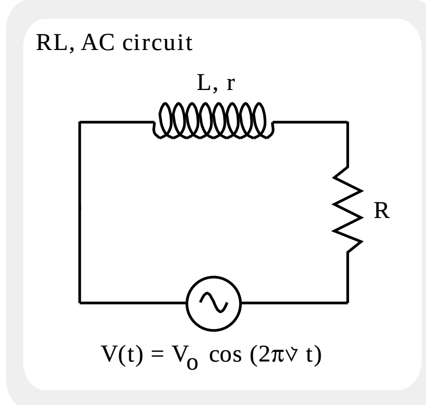

FIGURE 20.9

2.) The inductor in the circuit will have a certain amount of resistance r_L inherent within the wires that make up its coils. That resistance will act like any other resistor-like element in an AC circuit.

3.) In addition to r_L, the inductor also has a resistive nature that is <u>frequency-dependent</u>. Not obvious? Follow along.

a.) When an alternating current passes through an inductor, Faraday's Law demands that an induced EMF be generated across the leads of the coil that will ultimately produce an induced magnetic flux that opposes the changing magnetic flux through the coil's face. From the previous chapter, the magnitude of this induced EMF equals:

$$\varepsilon_L = L\,(di/dt).$$

b.) Writing a Loop Equation for an RL circuit (see Figure 20.10 for the voltages associated with each element), we get:

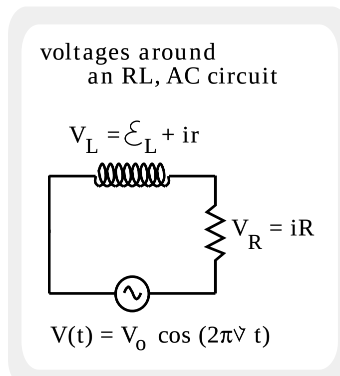

FIGURE 20.10

$$(-\varepsilon_L - ir) - iR + V_o \cos(2\pi\nu t) = 0.$$

In this expression, ε_L is the induced, frequency-dependent voltage drop across the inductor (i.e., L di/dt), $2\pi\nu$ is the angular frequency of the power supply, and a cosine function has been used to characterize the varying voltage across the power supply (I've done this because it will make life easier when we do the evaluation that is to follow--we could as well have used a sine function, but the resulting expression would have been a bit messy). Substituting in and rearranging this expression, we get:

$$L\,(di/dt) + ir + iR = V_o \cos(2\pi\nu\, t).$$

c.) The resistor-like resistance inherent in an inductor is sometimes negligibly small, sometimes not, so for the sake of simplicity we will lump it with R to get R_{net}. Doing so, the above expression becomes:

$$L\,(di/dt) + iR_{net} = V_o \cos(2\pi\nu t).$$

d.) Though you will never have to derive this on a test, we need an expression for the resistive nature of the inductor excluding the resistor-like resistance inherent within its wires. To do this:

i.) Assume the resistance of (and, hence, voltage across) all of the resistor-like elements in the circuit is negligible (i.e., that $R_{net} = 0$). In that case, Kirchoff's Loop Equation becomes:

$$L\,(di/dt) = V_o \cos(2\pi\nu t).$$

ii.) We know the voltage across the power supply and inductor. We'd like an expression for the current through the circuit. To determine this, we need to manipulate and integrate. Doing so yields:

$$L\frac{di}{dt} = V_o\cos(2\pi\nu t)$$

$$\Rightarrow \quad L\,di = V_o\cos(2\pi\nu t)dt$$

$$\Rightarrow \quad L\int_{i=0}^{i} di = V_o\int_{t=0}^{t}\cos(2\pi\nu t)dt$$

$$\Rightarrow \quad Li = V_o\left(\frac{1}{2\pi\nu}\right)\left[\sin(2\pi\nu t)\right]$$

$$\Rightarrow \quad i = \frac{V_o\left[\sin(2\pi\nu t)\right]}{(2\pi\nu L)} \qquad \text{(Equation A).}$$

4.) Important point:

a.) Ohm's Law maintains that the current through an element must equal the voltage across the element divided by a quantity that reflects the resistive nature of the element. In the above expression, the voltage across the element is evidently $V_o[\sin(2\pi\nu t)]$. That means the resistive nature of the inductor must be $2\pi\nu L$.

b.) In fact, this is the frequency-dependent resistive nature of an inductor. It is called the inductive reactance, its symbol is X_L, and its units are ohms. Summarizing, we can write:

$$X_L = 2\pi\nu L \text{ (ohms)},$$

where the inductance L must be written in terms of henrys (versus milli-henrys or whatever).

c.) Although we assumed the resistor-like resistance of the circuit was negligible to do the derivation, in fact this inductive reactance expression is true whether the resistor-like resistance is big or small.

5.) Side-Note: Does the frequency-dependent expression for the resistive nature of an inductor (i.e., its inductive reactance) make sense? Consider:

a.) Consider a general RL circuit (i.e., one in which R_{net} is not small) hooked across a power supply that runs at low frequency.

Note: A low frequency voltage means that although the amplitude of the voltage of the power supply may be large or small, the rate at which the voltage changes is very slow.

i.) A low frequency voltage will produce a low frequency current.

ii.) A low frequency current means that di/dt will be small (the current is changing slowly if it is low frequency).

iii.) A small di/dt means the induced voltage drop across the inductor (Ldi/dt) is small.

iv.) A small induced voltage drop across the inductor implies a relatively large voltage drop across the resistor (at any instant, the two have to add up to the voltage across the power supply--a quantity that can be large).

v.) As the voltage drop across a resistor is directly proportional to the current through the resistor, a large voltage drop across the resistor implies a relatively large current through the resistor and, hence, through the circuit.

b.) <u>Bottom line #1</u>: The current in an RL circuit will be relatively large when a low frequency signal passes through the circuit. That means we would expect the inductive reactance (the resistive nature of the inductor) to be small at low frequencies. This is exactly what our derived expression predicts (i.e., when ν is small, $X_L = 2\pi\nu L$ is small).

c.) Using similar reasoning, a power supply running at high voltage creates a high frequency current that will produce a very large di/dt value. In such a case, the voltage drop across the inductor (L di/dt) is relatively large and the voltage drop across the resistor is relatively small. A small voltage drop across the resistor suggests a small current flowing in the circuit.

d.) Bottom line #2: The current in an RL circuit will be relatively small (i.e., approaching zero) when a high frequency signal passes through the circuit. That means we would expect the inductive reactance to be big at high frequencies. This is exactly what our expression predicts (i.e., when ν is large, X_L is large).

e.) Summary: An inductor in an AC circuit passes low frequency signals while damping out high frequency signals. As such, inductors are sometimes referred to as low pass filters.

6.) The second point to note about Equation A again has to do with its form. By assuming a power supply voltage that is proportional to cos $(2\pi\nu t)$, and assuming that the net resistance in the circuit is zero (i.e., $R_{net} = 0$ so that the voltage across the inductor is the same as that across the power supply), we find that the circuit's current is proportional to sin $(2\pi\nu t)$. Examining the graph of these two functions (the current is shown in Figure 20.11a and the voltage shown in Figure 20.11b) allows us to conclude that in this situation the voltage across the inductor leads the current through the inductor (i.e., the circuit's current) by $\pi/2$ radians.

Note: This $\pi/2$ phase shift exists ONLY if there is no resistor-like resistance in the circuit. As there will never be the case in which there is absolutely no resistor-like resistance in a circuit, the phase shift in a real AC circuit will never by $\pi/2$. Calculating the real shift is something you will run into shortly.

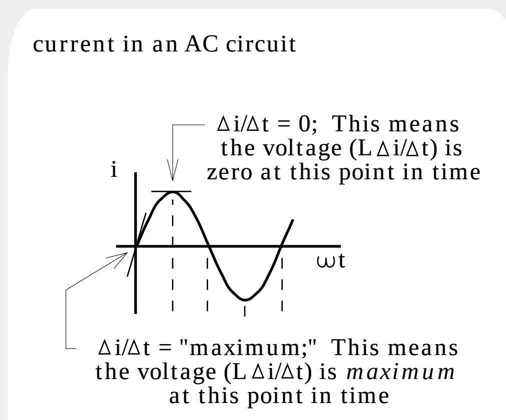

FIGURE 20.11a

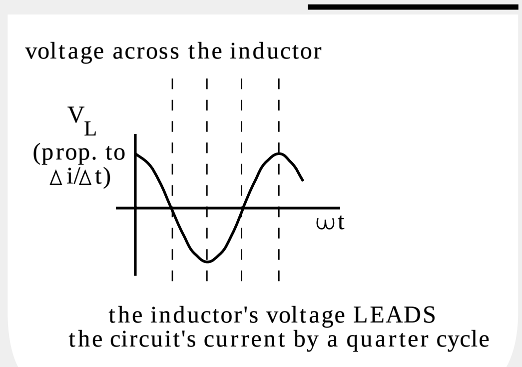

FIGURE 20.11b

F.) A Capacitor and Resistor in an AC Circuit:

1.) Consider the RC circuit shown in Figure 20.12.

2.) Unless it is leaky, the capacitor in the circuit will have no resistor-like resistance inherent within it. As such, we will assume there is no ir voltage drop across the capacitor.

3.) Just as there is a frequency-dependent resistive nature associated with an inductor, there is a frequency-dependent resistive nature associated with a capacitor. To see this, consider the following:

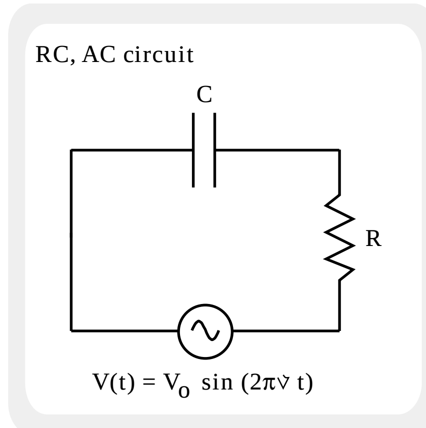

FIGURE 20.12

a.) The voltage drop across a capacitor is defined as:

$$V_C = q/C,$$

where q is the magnitude of charge on one capacitor plate and C is the capacitor's capacitance.

b.) In this case, again to make the evaluation easier later on, let's assume the power supply's voltage is characterized as a sine function (a cosine function would also work, but it would be a little messy to make sense of later, so we'll use the sine). With that assumption, a Kirchoff's Loop Equation for this circuit (see Figure 20.13) becomes:

$$- q/C - iR + V_o \sin (2\pi\nu t) = 0.$$

Manipulating, we get:

$$q/C + iR = V_o \sin (2\pi\nu t),$$

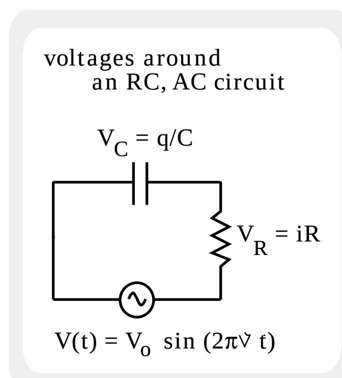

FIGURE 20.13

where q is a time varying quantity in the expression (we could denote it q(t), but, for simplicity, we will leave it as presented).

c.) Though you will never have to derive this on a test, we need an expression for the resistive nature of the capacitor. To do this:

i.) Assume the resistor-like resistance in the circuit is negligible (i.e., that R = 0). In that case, Kirchoff's Law becomes:

$$q/C = V_o \sin(2\pi\nu t)$$
$$\Rightarrow \quad q = CV_o \sin(2\pi\nu t).$$

ii.) Remembering that i = dq/dt, we can write:

$$\begin{aligned} i &= \frac{dq}{dt} \\ &= \frac{d\left[CV_o \sin(2\pi\nu t)\right]}{dt} \\ &= CV_o(2\pi\nu)\cos(2\pi\nu t) \\ &= \frac{V_o \cos(2\pi\nu t)}{\left(\frac{1}{2\pi\nu C}\right)} \qquad \text{(Equation B).} \end{aligned}$$

4.) As pointed out in the inductor section, Ohm's Law maintains that the current through an element must equal the voltage across the element divided by a quantity that reflects the resistive nature of the element. In the above expression, the voltage across the element is $V_o\cos(2\pi\nu t)$. That means the resistive nature of the capacitor must be $1/(2\pi\nu C)$.

In fact, this is the frequency-dependent resistive nature of a capacitor. It is called the capacitive reactance, its symbol is X_C, and its units are ohms. Summarizing, we can write:

$$X_C = \frac{1}{2\pi\nu C} \quad \text{(ohms)},$$

where the capacitance C must be written in terms of farads (versus leaving it in microfarads or whatever).

5.) Side-Note: Does the frequency-dependent expression for the resistive nature of a capacitor (i.e., its capacitive reactance) make sense? Consider:

a.) Assume the voltage of a power supply runs at low frequency.

i.) Examining the low frequency signal shown in Figure 20.14a, it is evident that the signal is changing very slowly and that there is a respectable amount of charge on the capacitor a fair portion of the time. In other words, the capacitor has plenty of time to charge up and, on the average, the voltage (q/C) across the capacitor is relatively large.

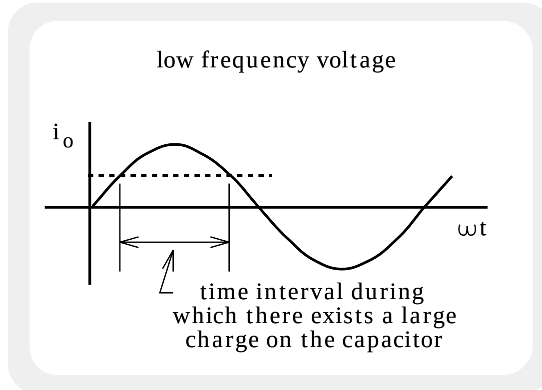

FIGURE 20.14a

ii.) Because the capacitor's voltage is relatively large on average, the voltage across the resistor will be relatively small. This implies a small current in the circuit.

b.) Bottom line #1: The current in an RC circuit will be relatively small (i.e., zero) when a low frequency signal passes through the circuit. That means we would expect the capacitive reactance (the resistive nature of the capacitor) to be large at low frequencies. This is exactly what our expression predicts (i.e., when ν is small, $X_C = 1/(2\pi\nu C)$ is large).

c.) Assume the voltage of a power supply now runs at high frequency.

i.) Examining the high frequency signal shown in Figure 20.14b, it is evident that the signal is changing very fast. There are not great spans of time during which the capacitor is charged, hence there are not great spans of time during which the voltage across the capacitor is high. In fact, the voltage (on the average) is low (remember, the time average of a high frequency sine wave is zero even over relatively small time intervals).

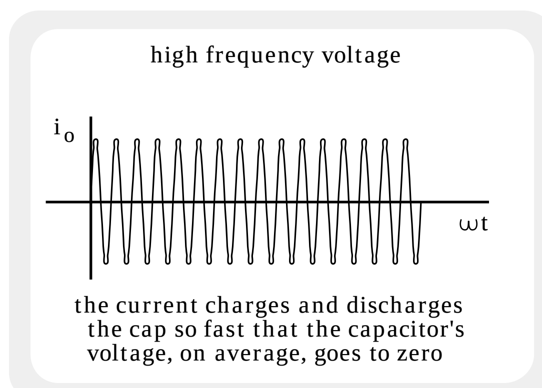

FIGURE 20.14b

ii.) A small voltage across the capacitor (on average) means a large voltage across the resistor. This implies a large current in the circuit.

d.) Bottom line #2: The current in an RC circuit will be relatively large when a high frequency signal passes through the circuit. That means we would expect the capacitive reactance to be small at high frequencies. This is exactly what our expression predicts (i.e., when ν is large, X_C is small).

e.) Summary: A capacitor in an AC circuit passes high frequency signals while damping out low frequency signals. As such, capacitors are sometimes referred to as high pass filters.

6.) The second point to note about Equation B is, again, its form. By assuming a power supply voltage that is proportional to sin $(2\pi\nu t)$, and assuming that the net resistance in the circuit is zero (i.e., $R_{net} = 0$ so that the voltage across the inductor is the same as that across the power supply), we find that the circuit's current is proportional to cos $(2\pi\nu t)$. Examining the graph of these two functions allows us to conclude that in this situation the voltage across the capacitor lags the current through the capacitor (i.e., the circuit's current) by $\pi/2$ radians.

Does this make sense?

a.) The voltage across a capacitor is proportional to the charge on the capacitor (i.e., $V_C = q/C$). Figure 20.15a depicts a graphical representation of this.

b.) Current is defined as the amount of charge that passes a particular point per unit time (i.e., $i = dq/dt$).

c.) The slope of the capacitor's voltage function is

$$dV_C/dt = (1/C)(dq/dt) = i/C.$$

d.) In other words, a graph of the slope of the capacitor's voltage function gives us a modi-

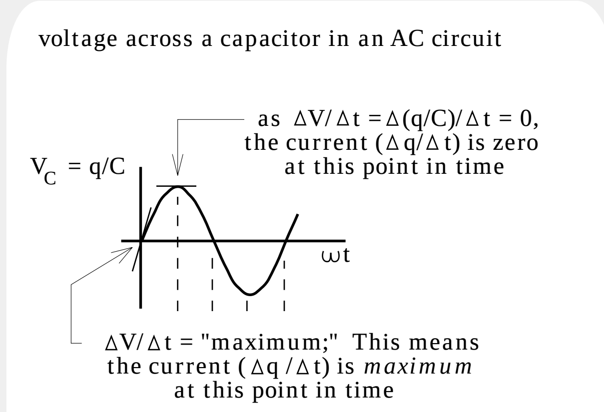

FIGURE 20.15a

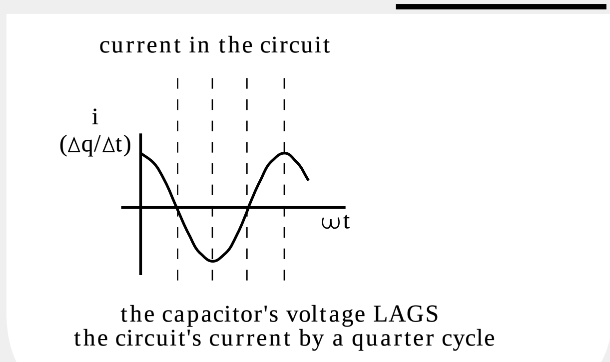

FIGURE 20.15b

fied current function. Figure 20.15b shows this.

e.) In comparing the graphs, it is evident that the voltage across the capacitor LAGS the current in the circuit by one quarter of a cycle, or $\pi/2$ radians.

Note: As was the case with the inductor, this $\pi/2$ phase shift exists ONLY if there is no resistor-like resistance in the circuit. As there will never be the case in which there is absolutely no resistor-like resistance in a circuit, the phase shift in a real RC circuit will never by $\pi/2$. Calculating what it actually is in a given case is something you will run into shortly.

G.) A Capacitor, Inductor, and Resistor in an AC Circuit:

1.) Until now, the algebraic expressions defining the resistive natures of capacitors and inductors have been perfectly straightforward. However, something a little more complex happens when a capacitor, inductor, and resistor are all placed in the same AC circuit (see Figure 20.16). As capacitors dampen out low frequency while inductors dampen out high frequency, the question arises, "Is there any frequency at which a current can flow?"

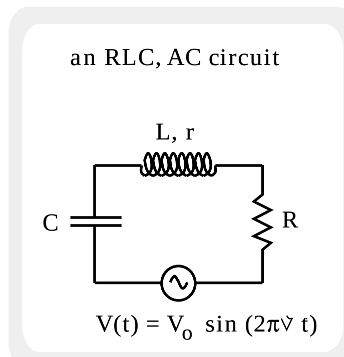

FIGURE 20.16

It turns out that there is one frequency (or a small range of frequencies) where the effect of the capacitor is negated by the effect of the inductor. At that frequency, the circuit's net resistive nature becomes small and a relatively large current flows. To determine that frequency, we must first determine an expression for the net resistive nature of the RLC circuit. To do that, we could do as we did with the RC and RL circuits. That is, we could write out Kirchoff's Loop equation for the circuit, solve for the current, then compare that expression with the Ohm's Law relationship i = V/(resistive nature) to determine the circuit's net resistive nature.

As mathematically intriguing as this might be, there is another way utilizing what are called phasor diagrams.

2.) Phasor diagrams:

a.) For a given frequency, there are three vectors in a phasor diagram:

i.) The magnitude of the first vector is equal to the magnitude of the net resistive nature provided by the elements in the circuit that do not throw the voltage out of phase with the current at all. These elements are resistors. Their net resistive nature is denoted by R_{net} (in most cases, this $R_{net} = R + r_L$--remember, there is usually resistor-like resistance inherent in the wire making up the inductor coil). This vector will be graphed along the +x axis.

Note: In this kind of diagram, circuit elements that leave the voltage in phase with the current are graphed BY DEFINITION along the +x axis. Elements that make the voltage lead the current by a quarter cycle are graphed at $+\pi/2$ radians from the +x axis (i.e., along the +y axis), and elements that make the voltage lag the current by a quarter cycle are graphed at $-\pi/2$ radians (i.e., along the -y axis).

ii.) The magnitude of the second vector is equal to the magnitude of the net resistive nature provided by the elements in the circuit that make the voltage lead the current by $\pi/2$ radians. These elements are the inductors. Their net resistive nature is denoted by X_L and that quantity is frequency-dependent. This vector will be graphed along the +y axis.

iii.) The magnitude of the third vector is equal to the magnitude of the net resistive nature provided by the elements in the circuit that make the voltage lag the current by $\pi/2$ radians. These elements are capacitors. Their net resistive nature is denoted by X_C and that quantity is frequency-dependent. This vector will be graphed along the -y axis.

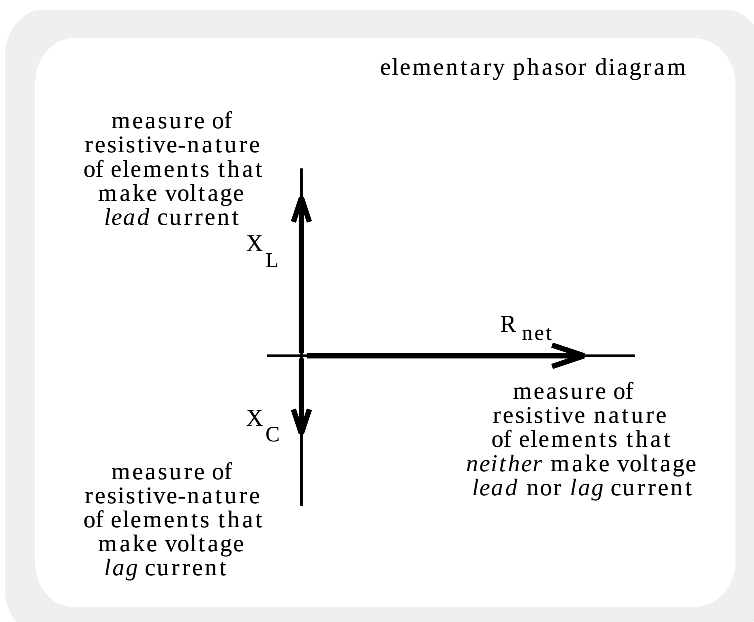

FIGURE 20.17a

b.) A typical phasor diagram is shown in Figure 20.17a. The vector addition of the two vectors found along the y axis is shown in Figure 20.17b, and the final vector addition (i.e., the one that produces the resultant Z) is shown in Figure 20.17c.

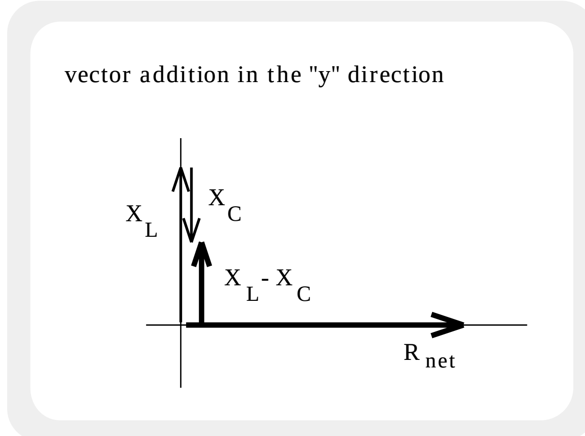

FIGURE 20.17b

3.) Impedance:

a.) The resultant of the phasor diagram is given a special name--the circuit's impedance (its symbol is Z). It tells us two things: the net resistive nature of the entire circuit and the phase shift of the circuit (i.e., the degree to which the voltage leads or lags the current at a given frequency).

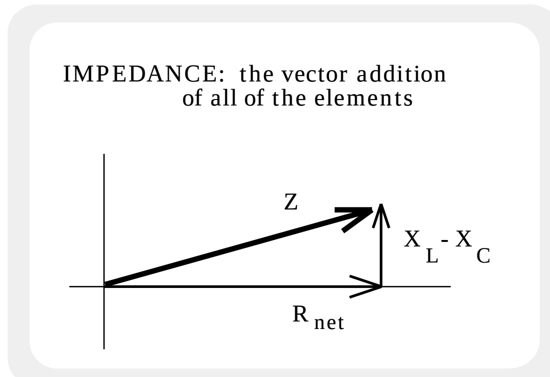

FIGURE 20.17c

b.) Algebraically (using the Pythagorean relationship and the diagram), the magnitude of a circuit's impedance equals:

$$Z = [R_{net}^2 + (X_L - X_C)^2]^{1/2}.$$

c.) The units of Z are ohms.

d.) Writing Z out in expanded form, we find that it is frequency-dependent:

$$Z = \left[R_{net}^2 + [2\pi\nu L - 1/(2\pi\nu C)]^2\right]^{1/2}.$$

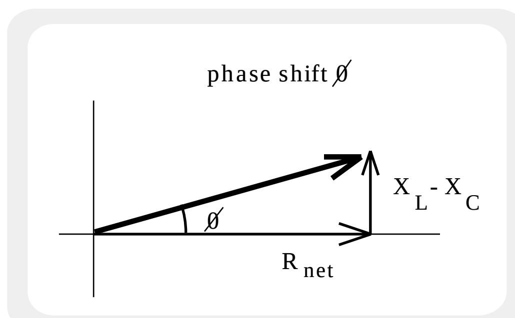

FIGURE 20.17d

e.) Using trig. and the phasor diagram shown in Figure 20.17d, the phase shift is found to be:

$$\phi = \tan^{-1} [(X_L - X_C) / R_{net}].$$

Note: If the calculated phase shift is positive, the voltage leads the current. If the phase shift is negative, the voltage lags the current.

4.) The Resonance Frequency of an RLC circuit: We began this section on RLC circuits by posing a question: "Is there a frequency at which the resistive effects of the inductor and capacitor cancel one another out leaving us with a substantial current flowing in the circuit?"

Our impedance expression gives us the answer.

a.) As both X_L and X_C are frequency-dependent, there must be a frequency at which $X_L - X_C$ equals zero. At that frequency, the net resistive quality of the circuit (i.e., its impedance) will be at a minimum, and a relatively sizable current should flow.

This special frequency is called the resonance frequency ν_{res}.

b.) Mathematically, the resonance frequency can be found as follows: At resonance,

$$X_L - X_C = 0$$

$$\Rightarrow \quad 2\pi\nu_{res}L - 1/(2\pi\nu_{res}C) = 0$$

$$\Rightarrow \nu_{res} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}.$$

A graph of the frequency response (i.e., current as a function of frequency) of an RLC circuit is shown in Figure 20.18.

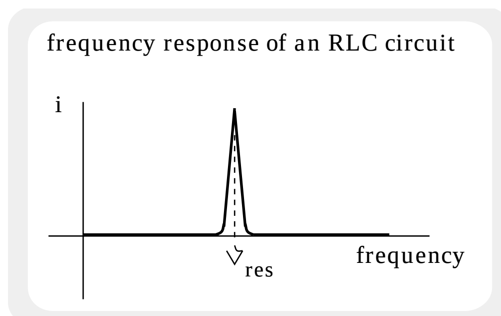

FIGURE 20.18

5.) One might wonder what is really happening at resonance. A more concise explanation follows:

a.) Assume a capacitor in an LC circuit is initially charged (see Figure 20.19a--there is resistance in the circuit but we will ignore it for now). There is no power supply in the circuit, so when the switch is thrown at t = 0 the capacitor begins to discharge through the inductor.

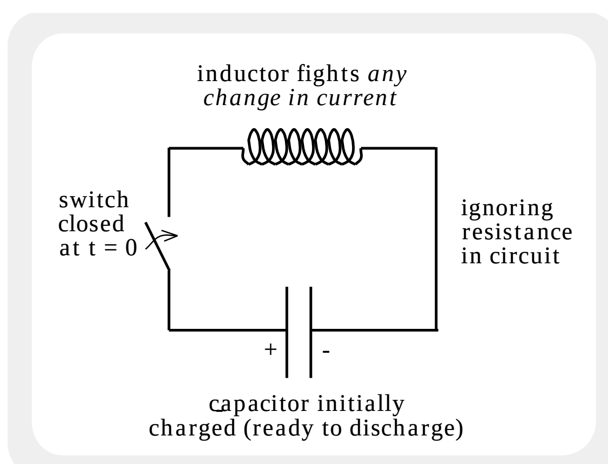

FIGURE 20.19a

b.) As a coil, the inductor produces a back-EMF which fights the increase of current in the circuit. As such, the current rises more slowly than would otherwise have been the case (though it does still rise).

c.) Sooner or later, the free charge on the discharging capacitor begins to run out. As this occurs, the current in the circuit declines prompting the inductor to produce an induced EMF that fights that change.

d.) Bottom line: Charge continues to flow in the circuit even after the capacitor is completely discharged.

e.) As current continues to flow, the capacitor begins to recharge going the other way (i.e., what was the positive terminal becomes the negative terminal and vice versa). When the induced current finally dies out, we find ourselves with a charged capacitor all ready to discharge, starting the whole process over again.

f.) If there is no resistance in the circuit (i.e., if the system's wires are superconductors), this oscillating AC charge flow will continue forever. With resistance in the circuit, the amplitude of the current will diminish with time but the charge/discharge frequency will not change (see Figure 20.19b).

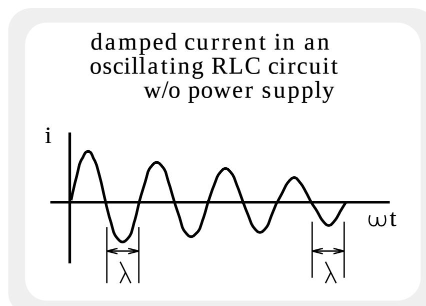

FIGURE 20.19b

g.) The natural frequency of this discharge, charge, discharge process is related to the size of the inductor and capacitor. Specifically, the system's natural frequency is $\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$.

h.) It shouldn't be hard to imagine what will happen if a power supply is placed in a circuit. If the frequency of the power supply does not match the natural frequency of the (R)LC combination, the charge/discharge frequency will fight the alternative voltage provided by the out-of-step power supply, and the net current in the circuit will be very small if not zero.

On the other hand, if the frequency of the power supply is just right (i.e., $\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$), its voltage will resonate with the charge/discharge cycle and the amplitude of the net current in the circuit will grow large. That frequency is the resonance frequency for the (R)LC circuit, and at that frequency the current will be as large as it ever can be.

H.) Summary of Circuit Elements in AC Circuits:

1.) The following is a chart devoted to presenting the information outlined above in an easily digested format. KNOW WHAT THE SYMBOLS MEAN--simply memorizing relationships is NOT going to help you on your next test!

element	symbl	units	resistive nature	phase	filter
resistor	R	ohms	resistance R (ohms)	no phase shift	none
inductor	L	henrys	inductive reactance: $X_L = 2\pi\nu L$ (ohms)	V_L leads circuit current by $\pi/2$ radians	low pass
capacitor	C	farads	capacitive reactance: $X_C = 1/(2\pi\nu C)$ (ohms)	V_C lags circuit current by $\pi/2$ radians	high pass
RLC circuit	--- --- ---	--- --- ---	impedance: $Z = [R^2 + (X_L - X_C)^2]^{1/2}$ (ohms)	phase shift (ϕ) $= \tan^{-1}[(X_L - X_C)/R]$ (radians)	re-son-ance frequ.

Note: Resonance frequency is $\nu_{res} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ hertz.

I.) Impedance Matching and Transformers:

1.) Optical light passing through an interface (i.e., a boundary) between two media will normally experience partial reflection caused by the fact that the two media have different densities. The only time a light beam will not reflect is when it passes from one medium into a second medium whose "density environment" is exactly the same as the first.

2.) Consider any complex electrical circuit--say, a stereo system connected to speakers. If the impedance of the stereo and the impedance of the collective speakers is the same, the signal will pass from the one to the other just as light passes through two common-density environments (i.e., there will be no reflection at the interface between the two systems). If, on the other hand, the resistive nature of the circuitry from which the signal comes (i.e., the stereo) is different from the resistive nature into which the signal must go (i.e., the speakers), reflection will occur at the interface. Put another way, maximum power will be transferred from the stereo to the speakers when the impedance of both is the same.

3.) The problem frequently confronting circuit designers is the fact that stereo systems have large impedances whereas speaker circuits have only tiny impedances. The question is, "How does one trick the signal into thinking the circuit it is entering has the same impedance as the circuit it is leaving?"

The answer involves the use of a transformer and is wrapped up in what is called impedance matching.

4.) A quick review of transformers: A transformer is essentially a pair of coils linked via a common magnetic field and, hence, a common magnetic flux. The turns-ratio (N_s/N_p) dictates how the secondary's voltage and current is related to the primary's voltage and current. That is:

a.) $N_s/N_p = \mathcal{E}_s/\mathcal{E}_p = i_p/i_s$ (this is true in all cases). Additionally;

b.) If $N_s < N_p$, the secondary's voltage is smaller than the primary's voltage ($\mathcal{E}_s < \mathcal{E}_p$) and the transformer is called a step-down transformer. In step-down transformers, the current in the secondary is larger than the current in the primary (i.e., $i_s > i_p$).

c.) If $N_s > N_p$, the secondary's voltage is larger than the primary's voltage ($\mathcal{E}_s > \mathcal{E}_p$) and the transformer is called a step-up transformer. In step-up transformers, the current in the secondary is smaller than the current in the primary (i.e., $i_s < i_p$).

5.) Having so reviewed, consider the following situation. A 1200 Ω stereo system (Z_{st} = 1200 Ω) is hooked up to a set of 8 Ω speakers (Z_{sp} = 8 Ω). From the signal's standpoint, how can we use a transformer to make 8 Ω speakers look like 1200 Ω elements?

a.) The first thing to notice is that as the signal comes in to the transformer, it sees a net impedance ($Z_{transf.+load}$) made up of the primary coil's impedance, the secondary coil's impedance, and the load's impedance (Z_{load}). This net impedance is what we want to numerically equal the stereo's impedance (Z_{st}). Put another way, the signal sees an entire package which, if the transformer system has been designed optimally, will appear to have an impedance of 1200 Ω.

b.) We know that the primary coil's current i_p will be the current coming from the stereo (i.e., $i_p = i_{stereo}$) while the primary coil's voltage is some value V_p. From Ohm's Law, the impedance of the stereo circuit (Z_{st}) will be:

$$Z_{st} = V_p/i_p \quad (= 1200\ \Omega \text{ for our example}).$$

c.) As the current from the stereo is i_p and the impedance of the stereo is Z_{st}, the energy provided by the stereo to the primary coil will be:

$$P_p = i_p{}^2 Z_{st}.$$

d.) Assuming an ideal transformer, the power provided by the primary will be completely transferred to the secondary. That is:

$$P_p = P_s.$$

e.) We know that the energy provided to the secondary circuit (i.e., the power available to the secondary circuit from the primary coil) will be dissipated by the load (the speakers). That is:

$$P_s = i_s{}^2 Z_{load}.$$

f.) Equating the power terms yields:

$$P_p = P_s$$
$$i_p{}^2 Z_{st} = i_s{}^2 Z_{load}$$
$$\Rightarrow \quad Z_{st} = (i_s{}^2 / i_p{}^2)\, Z_{load} \qquad \text{(Equation A).}$$

g.) We have already established the relationships that exist between the secondary and primary currents and the turns-ratio of the transformer. Specifically, we know that:

$$N_p/N_s = i_s/i_p.$$

h.) Using this to eliminate the current terms in Equation A leaves us with:

$$Z_{st} = (N_p{}^2 / N_s{}^2)\, Z_{load}.$$

i.) What does this relationship mean? Z_{st} and Z_{load} are fixed. Evidently, for the signal to transfer without reflection, the turns-ratio of the transformer must be such that:

$$(N_p / N_s)^2 = Z_{st} / Z_{load}.$$

j.) <u>Bottom line</u>: To modify the speaker-load to suit the incoming signal (i.e., to impedance match), we must use a transformer whose turns-ratio is such that:

$$(N_p / N_s)^2 = Z_{st} / Z_{load}.$$

where Z_{load} is the true load resistance (i.e., that of the speakers) and Z_{st} is the impedance of the signal's source.

k.) For our situation, $N_p/N_s = (1200\ \Omega/8\ \Omega)^{1/2} = 12.24/1$. If the winds are $N_p = 1224$ and $N_s = 100$, the signal will see the load as 1200Ω, and no reflection will occur.

QUESTIONS

20.1) An AC voltage source is found to produce a 12 volt peak to peak signal at 2500 hertz.

a.) Characterize this voltage as a sine function.

b.) Determine the RMS voltage of the source.

c.) It is found that when a capacitor and resistor are placed across the source as characterized above, an ammeter in the circuit reads 1.2 amps. What is the maximum current drawn from the source?

20.2) An RC circuit is hooked across an AC power supply. Which of the following statements are true (there can be more than one)? Explain each response.

a.) The RMS voltage across the resistor is the same as the average voltage across the resistor.

b.) The RMS voltage across the resistor is equal to R times the RMS current through the resistor.

c.) The RMS voltage across the resistor will be very large if the capacitive reactance is very large.

d.) The RMS current in the circuit will be very large if the capacitive reactance is very small.

e.) A decrease in frequency will increase the voltage across the capacitor.

f.) An increase in the capacitance will increase the current in the circuit for a given frequency.

g.) A decrease in frequency will increase the voltage across the resistor.

20.3) An RL circuit is hooked across an AC power supply. Assuming the inductor's internal resistance r_L equals zero, which of the following statements are true (there can be more than one)?

a.) The RMS voltage across the resistor is equal to R times the RMS current through the resistor.

b.) The RMS voltage across the resistor R will be very large if the inductive reactance is very large.

c.) The RMS current in the circuit will be very large if the inductive reactance is very small.

d.) A decrease in frequency will increase the voltage across the inductor.

e.) An increase in the inductance will increase the current in the circuit for a given frequency.

f.) A decrease in frequency will increase the voltage across the resistor.

20.4) Considering the circuit shown in Figure I, which of the resistors will dissipate most of the power (that is, energy) at a very high frequency?

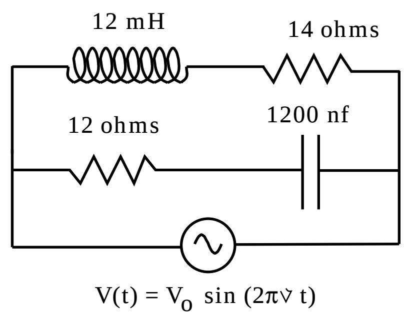

FIGURE I

20.5) A circuit has 220 ohms of impedance when a 95 ohm resistor is connected in series with a capacitor and a power source (V_{max} = 80 volts) supplying power at 300 hertz.

a.) What is the capacitive reactance at this frequency?

b.) Determine the circuit's impedance at 1000 hertz.

c.) Determine the circuit's RMS current at 1000 hertz.

20.6) A circuit driven at 240 hertz has 60 ohms of impedance when a 12 ohm resistor is connected in series with an inductor whose internal resistance is 8 ohms. The circuit's power source provides 70 volts RMS.

a.) What is the inductive reactance at this frequency?

b.) Determine the circuit's impedance at 1000 hertz.

c.) Determine the circuit's RMS current at 1000 hertz.

20.7) An RLC circuit incorporates a 12 ohm resistor, a 60 mH inductor, and a 12 μf capacitor hooked in series across a power supply whose time-dependent voltage is 140 sin (1100 t).

a.) Determine the frequency of the power supply's signal.

b.) Determine the capacitive reactance at this frequency.

c.) Determine the inductive reactance at this frequency.

d.) Determine the impedance of the circuit at this frequency.

e.) Determine the phase shift at this frequency. Is the voltage leading or lagging the current?

f.) Determine the power supply's RMS voltage.

g.) Determine the RMS current in the circuit at this frequency.

h.) Determine the resonance frequency for this circuit.

i.) Write out the time-dependent power supply voltage function at resonance.

j.) Determine the impedance of the circuit at resonance.

k.) Determine the RMS current in the circuit at resonance.

20.8) A transformer has 200 turns in its primary coil. A radio circuit has 237 ohms of impedance. Its signal has to be played through a 12 ohm speaker.

a.) Impedance match the two circuits. That is, explain how you would effect the match and give any pertinent numbers.

b.) Did you use a step-down or step-up transformer?